Kuwait University | Math 101 | May 15th, 2003
Math. & Comp. Sci. Dept. | Second Exam | Time: 75 minutes

Calculators, Mobile Phones, Pagers and all other mobile communication equipment are not allowed.

1. Let $f(x) = \sqrt{x-3}$. Use differentials to approximate $f(11.7)$.

(4 Points)

2. Find an equation of the normal line to the graph of $xy + (x+y)^3 + 1 = 0$ at $x = 0$.

(4 Points)

3. Find two real numbers x and y such that: $x + y = 16$ and $P = xy^3$ is maximum.

(4 Points)

4. Let f be a differentiable function on [1,3] with $f(1) = 3$ and $f(3) = 1$. Show that the graph of f admits a tangent line at $c \in (1,3)$ parallel to the line of the equation: $x + y - 4 = 0$.

(4 Points)

5. Let $f(x) = \dfrac{x}{(x+1)^2}$.

   a) Find the vertical and horizontal asymptotes (if any).

   b) Show that $f'(x) = \dfrac{1-x}{(x+1)^3}$. Find the intervals on which f is increasing and the intervals on which f is decreasing and then find the local extrema of f (if any).

   c) Given that $f''(x) = \dfrac{2(x-2)}{(x+1)^4}$. Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the point of inflection (if any).

   d) Sketch the graph of f.

(9 Points)

$f(x) = \sqrt{x-3}$, $f'(x) = \frac{1}{2\sqrt{x-3}}$.

a) $f(x) \cong f(x_0) + f'(x_0)(x - x_0) \Rightarrow \boxed{\sqrt{x-3} \cong \sqrt{x_0-3} + \frac{1}{2\sqrt{x_0-3}}(x-x_0)}$

b) $f(11.7) = ?$, put $x = 11.7$, $x_0 = 12$ $\Rightarrow$

$\sqrt{8.7} \cong \sqrt{9} + \frac{1}{2\sqrt{9}}(11.7 - 12) = \boxed{2.95.}$

2. $x=0 \Rightarrow y=-1$, $xy' + y + 3(x+y)^2(1+y') = 0 \Rightarrow y'\Big|_{(0,-1)} = -2/3$.

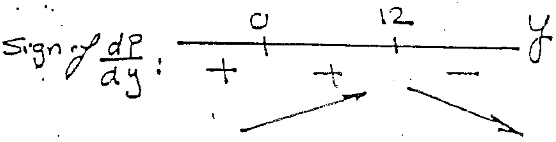
∴ The equation of the normal line is $\frac{y+1}{x-0} = 3/2$

$\Rightarrow \boxed{y = 3/2\, x - 1.}$

4. $x + y = 16$. , $P = xy^3 \Rightarrow P = (16-y)y^3$

$\frac{dP}{dy} = 48y^2 - 4y^3$, $\frac{dP}{dy} = 0 \Rightarrow y = 0, 12$

$\boxed{y = 12 , x = 4}$

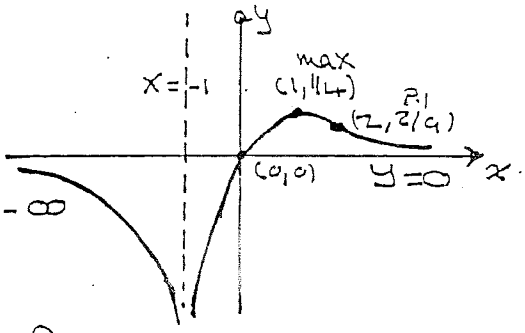


5. $x + y - 1 = 0 \Rightarrow y = -x + 4$ with slope -1.

$f'(c) = \frac{f(3) - f(1)}{3-1} \Rightarrow \boxed{f'(c) = -1}$

$f(x)$ is differentiable ⇒ Continuous ⇒ M.V.T. says that there is a number c in $(1,3)$ such that $f'(c) = -1$.

6. $f(x) = \frac{x}{(x+1)^2}$.



a) V.A: $\lim_{x \to -1^-} \frac{x}{(x+1)^2} = -\infty$, $\lim_{x \to -1^+} \frac{x}{(x+1)^2} = -\infty$

∴ $\boxed{x = -1}$ is a V.A.

H.A: $\lim_{x \to \pm\infty} \frac{x}{(x+1)^2} = 0$ ∴ $\boxed{y = 0}$ is a H.A.

b) $f'(x) = \frac{(x+1)^2 - x \cdot 2(x+1)}{(x+1)^4} = \frac{1-x}{(x+1)^3}$

$\boxed{f(1) = 1/4}$ is a local maximum.

Intervals	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Sign of f'	−	+	−
inc. & dec	↘	↗	↘

c) $f''(x) = \frac{(x+1)^3(-1) - (1-x)3(x+1)^2}{(x+1)^6} = \frac{2(x-2)}{(x+1)^4}$

∴ $\boxed{(2, 2/9)}$ is a point of inflection

Intervals	$(-\infty,-1)$	$(-1,2)$	$(2,\infty)$
Sign of f''	−	−	+
Concavity	∩	∩	∪